Which fractions become terminating decimals?

When you look at a common fraction, like $\frac{1}{2}$ or $\frac{5}{6}$, can you tell right away whether or not it will have a terminating decimal expansion or a repeating decimal expansion?

The goal of this problem sequence is that, by the end of it, you can!

To develop and test the conjectures we made as a class, we will collect more data.

1) Find the decimal expansion of each fraction in the chart below. You may divide the work amongst the members of your group. Use the space below to do any long division.

Fraction	Expansion
$\frac{1}{2}$	
$\frac{1}{3}$	
$\frac{1}{4}$	
$\frac{1}{5}$	
$\frac{1}{6}$	
$\frac{1}{7}$	$0.\overline{142857}$
$\frac{1}{8}$	0.125
$\frac{1}{9}$	
$\frac{1}{10}$	
$\frac{1}{11}$	
$\frac{1}{12}$	
$\frac{1}{13}$	
$\frac{1}{14}$	
$\frac{1}{15}$	
$\frac{1}{16}$	
$\frac{1}{17}$	
$\frac{1}{18}$	
$\frac{1}{19}$	
$\frac{1}{20}$	

Fun fact: The bar above the digits $\overline{142857}$ means that these digits repeat forever like this: 0.142857142857142857142857…

Answer the following questions on a separate sheet of paper. Keep your work NEAT and ORGANIZED – trust me on this one!

2) Circle the fractions in the chart that have *terminating* decimal expansions. What else do they have in common? (HINT: look at their denominators.)

3) Based on your answer to question 2, guess three more fractions with terminating decimal expansions with denominators between 20 and 100. Turn them into decimals and check that they terminate. If they do, circle them.

4) Turn the following decimals into fractions:
   a. 0.1
   b. 0.33
   c. 0.541

5) Compare the denominators of the three fractions in problem 4 with the denominators of the fractions with terminating decimal expansions that you found before. What is common among all of them?

6) Turn these decimals into fractions with denominators of 10, 100, or 1000. Then simplify the fractions using ONLY *prime* factors until the fractions are completely reduced. *Keep track of the factors you divide by!*
   a. 0.075
   b. 0.4
   c. 0.06
   d. 0.175
   e. 0.45
   f. 0.176

7) a. What factors did you use when reducing the fractions in #6?
   b. What are the *prime* factors of the *reduced* denominators?
      (Hint/related questions to consider: Why only these factors? What are the prime factors of 10, 100, 1000?)

8) Now you might be able to answer the question that titles this problem set. Finish the following two sentences:
   a. The decimal expansion of a common fraction terminates if that common fraction…
   b. The decimal expansion of this type of fraction terminates because…